TABLE 34.9 Formulas for Computing Dedendum Angles and Their Sum

Type of taper	Formula
Standard	$\Sigma\delta = \tan^{-1}\dfrac{b_P}{A_m} + \tan^{-1}\dfrac{b_G}{A_m}$ $\delta_P = \tan^{-1}\dfrac{b_P}{A_m} \qquad \delta_G = \Sigma\delta - \delta_P$
Duplex	$\Sigma\delta = \dfrac{90[1 - (A_m/r_c)\sin\psi]}{(P_d A_o \tan\phi \cos\psi)}$ $\delta_P = \dfrac{a_G}{h}\Sigma\delta \qquad \delta_G = \Sigma\delta - \delta_P$
Tilted root line	Use $\Sigma\delta = \dfrac{90[1 - (A_m/r_c)\sin\psi]}{(P_d A_o \tan\phi \cos\psi)}$ or $= 1.3\tan^{-1}\dfrac{b_P}{A_m} + 1.3\tan^{-1}\dfrac{b_G}{A_m}$ whichever is smaller. $\delta_P = \dfrac{a_G}{h} \qquad \delta_G = \Sigma\delta - \delta_P$
Uniform depth	$\Sigma\delta = 0$ $\delta_P = \delta_G = 0$

34.5.4 AGMA References†

The following AGMA standards are helpful in designing bevel and hypoid gears:

AGMA Design Manual for Bevel Gears, 2005

AGMA Rating Standard for Bevel Gears, 2003

These are available through American Gear Manufacturer's Association, 1500 King Street, Suite 201, Alexandria, VA 22314-2730.

34.6 GEAR STRENGTH

Under ideal conditions of operation, bevel and hypoid gears have a tooth contact which utilizes the full working profile of the tooth without load concentration in any

† The notation and units used in this chapter are the same as those used in the AGMA standards. These may differ in some respects from those used in other chapters of this Handbook.

TABLE 34.10 Formulas for Computing Blank and Tooth Dimensions of Hypoid Gears

Item	No.	Formula
Pitch diameter of gear	1	$D = \dfrac{N}{P_d}$
	2	$m = \dfrac{n}{N}$
	3	$\psi_{Po} = \psi_P$
	4	$\Delta\Sigma = 90 - \Sigma$
	5	$\tan \Gamma_i = \dfrac{\cos \Delta\Sigma}{1.2(m - \sin \Delta\Sigma)}$
	6	$R = 0.5(D - F \sin \Gamma_i)$
	7	$\sin \varepsilon_i' = \dfrac{E}{R} \sin \Gamma_i$
	8	$K_1 = \tan \psi_{Po} \sin \varepsilon_i' + \cos \varepsilon_i'$
	9	$R_{P2} = mRK_1$
	10	$\tan \eta = \dfrac{E}{R(\tan \Gamma_i \cos \Delta\Sigma - \sin \Delta\Sigma) + R_{P2}}$ first trial
	11	$\sin \varepsilon_2 = \dfrac{E - R_{P2} \sin \eta}{R}$
	12	$\tan \gamma_2 = \dfrac{\sin \eta}{\tan \varepsilon_2 \cos \Delta\Sigma} + \tan \Delta\Sigma \cos \eta$
	13	$\sin \varepsilon_2' = \dfrac{\sin \varepsilon_2 \cos \Delta\Sigma}{\cos \gamma_2}$
	14	$\tan \psi_{P2} = \dfrac{K_1 - \cos \varepsilon_2'}{\sin \varepsilon_2'}$
	15	$\Delta K = \sin \varepsilon_2' (\tan \psi_{Po} - \tan \psi_{P2})$
	16	$\dfrac{\Delta R_P}{R} = m(\Delta K)$
	17	$\sin \varepsilon_1 = \sin \varepsilon_2 - \dfrac{\Delta R_P}{R} \sin \eta$
Pinion pitch angle	18	$\tan \gamma = \dfrac{\sin \eta}{\tan \varepsilon_1 \cos \Delta\Sigma} + \tan \Delta\Sigma \cos \eta$
	19	$\sin \varepsilon_1' = \dfrac{\sin \varepsilon_1 \cos \Delta\Sigma}{\cos \gamma_1}$
Pinion spiral angle	20	$\tan \psi_P = \dfrac{K_1 + \Delta K - \cos \varepsilon_1'}{\sin \varepsilon_1'}$

TABLE 34.10 Formulas for Computing Blank and Tooth Dimensions of Hypoid Gears (*Continued*)

Item	No.	Formula
Gear spiral angle	21	$\psi_G = \psi_P - \varepsilon_1'$
Gear pitch angle	22	$\tan \Gamma = \dfrac{\sin \varepsilon_1}{\tan \eta \cos \Delta\Sigma} + \cos \varepsilon_1 \tan \Delta\Sigma$
Gear mean cone distance	23	$A_{mG} = \dfrac{R}{\sin \Gamma}$
Pinion mean cone distance	24	$\Delta R_P = R\left(\dfrac{\Delta R_P}{R}\right)$
	25	$A_{mP} = \dfrac{R_{P2} + \Delta R_P}{\sin \gamma}$
	26	$R_P = A_{mP} \sin \gamma$
Limit pressure angle	27	$-\tan \phi_{01} = \dfrac{\tan \gamma \tan \Gamma}{\cos \varepsilon_1'} \times \dfrac{A_{mP} \sin \psi_P - A_{mG} \sin \psi_G}{A_{mP} \tan \gamma + A_{mG} \tan \Gamma}$
	28	$\text{Den} = -\tan \phi_{01}\left(\dfrac{\tan \psi_P}{A_{mP} \tan \gamma} + \dfrac{\tan \psi_G}{A_{mG} \tan \Gamma}\right) + \dfrac{1}{A_{mP} \cos \psi_P} - \dfrac{1}{A_{mG} \cos \psi_G}$
	29	$r_{c1} = \dfrac{\sec \phi_{01} (\tan \psi_P - \tan \phi_G)}{\text{Den}}$
	30	$\left\lvert \dfrac{r_c}{r_{c1}} - 1 \right\rvert \leq 0.01$ Loop back to no. 10 and change η until satisfied.
Gear pitch apex beyond crossing point	31	$Z_P = A_{mP} \tan \gamma \sin \Gamma - \dfrac{E \tan \Delta\Sigma}{\tan \varepsilon_1}$
	32	$Z = \dfrac{R}{\tan \Gamma} - Z_P$
Gear outer cone distance	33	$A_o = \dfrac{0.5D}{\sin \Gamma}$
	34	$\Delta F_o = A_o - A_{mG}$
Depth factor	35	k_1 (see Table 34.5)
Addendum factor	36	C_1 (see Table 34.7)
Mean working depth	37	$h = \dfrac{k_1 R \cos \psi_G}{N}$
Mean addendum	38	$a_P = h - a_G \qquad a_G = C_1 h$

TABLE 34.10 Formulas for Computing Blank and Tooth Dimensions of Hypoid Gears (*Continued*)

Item	No.	Formula
Clearance factor	39	k_2 (see Table 34-6)
Mean dedendum	40	$b_P = b_G + a_G - a_P \qquad b_G = h(1 + k_2 - C_1)$
Clearance	41	$c = k_2 h$
Mean whole depth	42	$h_m = a_G + b_G$
Sum of dedendum angle	43	$\Sigma\delta$ (see Sec. 34.5.2)
Gear dedendum angle	44	δ_G (see Sec. 34.5.2)
Gear addendum angle	45	$\alpha_G = \Sigma\delta - \delta_G$
Gear outer addendum	46	$a_{oG} = a_G + \Delta F_o \sin \alpha_G$
Gear outer dedendum	47	$b_{oG} = b_G + \Delta F_o \sin \delta_G$
Gear whole depth	48	$h_t = a_{oG} + b_{oG}$
Gear working depth	49	$h_k = h_{tG} - c$
Gear root angle	50	$\Gamma_R = \Gamma - \delta_G$
Gear face angle	51	$\Gamma_o = \Gamma + \alpha_G$
Gear outside diameter	52	$D_o = 2a_{oG} \cos \Gamma + D_G$
Gear crown to crossing point	53	$X_o = Z_P + \Delta F_o \cos \Gamma - a_{oG} \sin \Gamma$
Gear root apex beyond crossing point	54	$Z_R = Z + \dfrac{A_{mG} \sin \delta_G - b_G}{\sin \Gamma_R}$
Gear face apex beyond crossing point	55	$Z_o = Z + \dfrac{A_{mG} \sin \alpha_G - a_G}{\sin \Gamma_o}$
	56	$Q_R = \dfrac{A_{mG} \cos \delta_G}{\cos \Gamma_R} - Z$
	57	$Q_o = \dfrac{A_{mG} \cos \alpha_G}{\cos \Gamma_o} - Z$
	58	$\tan \xi_R = \dfrac{E \tan \Delta\Sigma}{Q_R}$

TABLE 34.10 Formulas for Computing Blank and Tooth Dimensions of Hypoid Gears (*Continued*)

Item	No.	Formula
Gear face apex beyond crossing point (*continued*)	59	$\tan \xi_o = \dfrac{E \tan \Delta\Sigma}{Q_o}$
	60	$\sin(\varepsilon_R + \xi_R) = \dfrac{E \cos \xi_R \tan \Gamma_R}{Q_R}$
	61	$\sin(\varepsilon_o + \xi_o) = \dfrac{E \cos \xi_o \tan \Gamma_o}{Q_o}$
Pinion face angle	62	$\sin \gamma_o = \sin \Delta\Sigma \sin \Gamma_R + \cos \Delta\Sigma \cos \Gamma_R \cos \varepsilon_R$
Pinion root angle	63	$\sin \gamma_R = \sin \Delta\Sigma \sin \Gamma_o + \cos \Delta\Sigma \cos \Gamma_o \cos \varepsilon_o$
Pinion face apex beyond crossing point	64	$G_o = \dfrac{E \sin \varepsilon_R \cos \Gamma_R - Z_R \sin \Gamma_R - c}{\sin \gamma_o}$
Pinion root apex beyond crossing point	65	$G_R = \dfrac{E \sin \varepsilon_o \cos \Gamma_o - Z_o \sin \Gamma_o - c}{\sin \gamma_R}$
	66	$\tan \lambda' = \dfrac{m \sin \varepsilon_i' \cos \Gamma}{\cos \gamma + m \cos \Gamma \cos \varepsilon_i'}$
Pinion addendum angle	67	$\alpha_P = \gamma_o - \gamma$
Pinion dedendum angle	68	$\delta_P = \gamma - \gamma_R$
Pinion whole depth	69	$h_{tP} = \dfrac{(x_o + G_o) \sin \delta_P}{\cos \gamma_o} - \sin \gamma_R (G_R - G_o)$
	70	$\Delta F_i = F - \Delta F_o$
	71	$\Delta F_{oP} = h \sin \varepsilon_R (1 - m)$
	72	$F_{oP} = \dfrac{\Delta F_o \cos \lambda'}{\cos(\varepsilon_i' - \lambda')}$
	73	$F_{iP} = \dfrac{\Delta F_i \cos \lambda'}{\cos(\varepsilon_i' - \lambda')}$
	74	$\Delta B_o = \dfrac{F_o \cos \gamma_o}{\cos \alpha_P} + \Delta F_{oP} - (b_G - c) \sin \gamma$
	75	$\Delta B_i = \dfrac{F \cos \gamma_o}{\cos \alpha_P} + \Delta F_{oP} - (b_G - c) \sin \gamma$
Pinion crown to crossing point	76	$x_o = \dfrac{E}{\tan \varepsilon_1 \cos \Delta\Sigma} - R_P \tan \gamma + \Delta B_o$

TABLE 34.10 Formulas for Computing Blank and Tooth Dimensions of Hypoid Gears (*Concluded*)

Item	No.	Formula
Pinion front crown to crossing point	77	$x_i = \dfrac{E}{\tan \varepsilon_1 \cos \Delta\Sigma} - R_P \tan \gamma - \Delta B_i$
Pinion outside diameter	78	$d_o = 2 \tan \gamma_o (x_o + G_o)$
Pinion face width	79	$F_P = \dfrac{x_o - x_i}{\cos \gamma_o}$
Mean circular pitch	80	$p_m = \dfrac{\pi A_{mG}}{P_d A_o}$
Mean diametral pitch	81	$P_{dm} = P_d \dfrac{A_o}{A_{mG}}$
Thickness factor	82	K (see Fig. 34-17)
Mean pitch diameter	83	$d_m = 2A_{mP} \sin \gamma$
	84	$D_m = 2A_{mG} \sin \Gamma$
Mean normal circular thickness	85	$t_n = p_m \cos \psi_G - T_n$
	86	$T_n = 0.5p_m \cos \psi_G - (a_P - a_G) \tan \phi + \dfrac{K \cos \psi}{P_{dm} \tan \phi}$
Outer normal backlash allowance	87	B (see Table 34-8)
Mean normal chordal thickness	88	$t_{nc} = t_n - \dfrac{t_n^3}{6d_m^2} - 0.5B \sec \phi \left(\dfrac{A_{mg}}{A_o}\right)$
	89	$T_{nc} = T_n - \dfrac{T_n^3}{6D_m^2} - 0.5B \sec \phi \left(\dfrac{A_{mG}}{A_o}\right)$
Mean chordal addendum	90	$a_{cP} = a_P + \dfrac{0.25t_n^2 \cos \gamma}{d_m}$
	91	$a_{cG} = a_G + \dfrac{0.25T_N^2 \cos \Gamma}{D_m}$

area. The recommendations and rating formulas which follow are designed for a tooth contact developed to give the correct pattern in the final mountings under full load.

34.6.1 Formulas for Contact and Bending Stress

The basic equation for contact stress in bevel and hypoid gears is

$$S_c = C_p \sqrt{\frac{2T_p C_o}{C_v} \frac{1}{FD^2} \frac{N}{n} \frac{1.2C_m C_f}{I}} \tag{34.1}$$

and the basic equation for bending stress is

$$S_t = \frac{2T_G K_o}{K_v} \frac{P_d}{FD} \frac{1.2K_m}{J} \tag{34.2}$$

where
S_t = calculated tensile bending stress at root of gear tooth, pounds per square inch (lb/in^2)
S_c = calculated contact stress at point on tooth where its value will be maximum, lb/in^2
C_p = elastic coefficient of the gear-and-pinion materials combination, (lb)$^{1/2}$/in
T_P, T_G = transmitted torques of pinion and gear, respectively, pound-inches (lb · in)
K_o, C_o = overload factors for strength and durability, respectively
K_v, C_v = dynamic factors for strength and durability, respectively
K_m, C_m = load-distribution factors for strength and durability, respectively
C_f = surface-condition factor for durability
I = geometry factor for durability
J = geometry factor for strength

34.6.2 Explanation of Strength Formulas and Terms

The elastic coefficient for bevel and hypoid gears with localized tooth contact pattern is given by

$$C_p = \sqrt{\frac{3}{2\pi} \frac{1}{(1-\mu_P^2)/E_P + (1-\mu_G^2)/E_G}} \tag{34.3}$$

where
μ_P, μ_G = Poisson's ratio for materials of pinion and gear, respectively (use 0.30 for ferrous materials)
E_P, E_G = Young's modulus of elasticity for materials of pinion and gear, respectively (use 30.0×10^6 lb/in^2 for steel)

The overload factor makes allowance for the roughness or smoothness of operation of both the driving and driven units. Use Table 34.11 as a guide in selecting the overload factor.

The dynamic factor reflects the effect of inaccuracies in tooth profile, tooth spacing, and runout on instantaneous tooth loading. For gears manufactured to AGMA class 11 tolerances or higher, a value of 1.0 may be used for dynamic factor. Curve 2 in Fig. 34.18 gives the values of C_v for spiral bevels and hypoids of lower accuracy or for large, planed spiral-bevel gears. Curve 3 gives the values of C_v for bevels of lower accuracy or for large, planed straight-bevel gears.

TABLE 34.11 Overload Factors K_o, C_o†

Prime mover	Character of load on driven member: Uniform	Medium shock	Heavy shock
Uniform	1.00	1.25	1.75
Medium shock	1.25	1.50	2.00
Heavy shock	1.50	1.75	2.25

†This table is for speed-decreasing drive; for speed-increasing drives add $0.01(N/n)^2$ to the above factors.

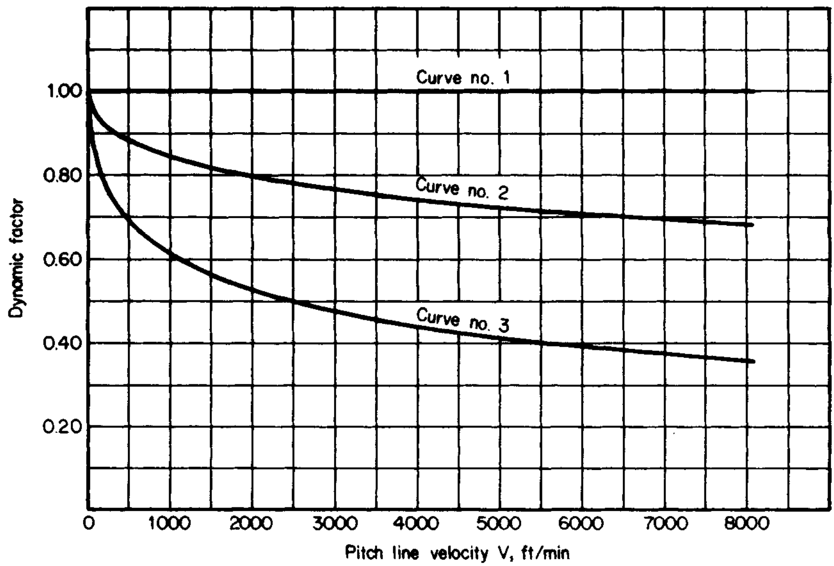

FIGURE 34.18 Dynamic factors K_v and C_v.

The load-distribution factor allows for misalignment of the gear set under operating conditions. This factor is based on the magnitude of the displacements of the gear and pinion from their theoretical correct locations. Use Table 34.12 as a guide in selecting the load-distribution factor.

The surface-condition factor depends on surface finish as affected by cutting, lapping, and grinding. It also depends on surface treatment such as lubrizing. And C_f can be taken as 1.0 provided good gear manufacturing practices are followed.

Use Table 34.13 to locate the charts for the two geometry factors I and J.

The geometry factor for durability I takes into consideration the relative radius of curvature between mating tooth surfaces, load location, load sharing, effective face width, and inertia factor.

The geometry factor for strength J takes into consideration the tooth form factor, load location, load distribution, effective face width, stress correction factor, and inertia factor.

TABLE 34.12 Load-Distribution Factors K_m, C_m

Application	Both members straddle-mounted	One member straddle-mounted	Neither member straddle-mounted
General industrial	1.00–1.10	1.10–1.25	1.25–1.40
Automotive	1.00–1.10	1.10–1.25	
Aircraft	1.00–1.25	1.10–1.40	1.25–1.50

TABLE 34.13 Location of Geometry Factors

				Figure no.	
Gear type	Pressure angle, ϕ	Shaft angle, Σ	Helix angle, ψ	I Factor	J Factor
Straight bevel	20°	90°	0°	34.19	34.20
	25°	90°	0°	34.21	34.22
Spiral bevel	20°	90°	35°	34.23	34.24
	20°	90°	25°	34.25	34.26
	20°	90°	15°	34.27	34.28
	25°	90°	35°	34.29	34.30
	20°	60°	35°	34.31	34.32
	20°	120°	35°	34.33	34.34
	20°†	90°	35°	34.35	34.36
Hypoid	19°	$E/D = 0.10$		34.37	34.38
	19°	$E/D = 0.15$		34.39	34.40
	19°	$E/D = 0.20$		34.41	34.42
	22½°	$E/D = 0.10$		34.43	34.44
	22½°	$E/D = 0.15$		34.45	34.46
	22½°	$E/D = 0.20$		34.47	34.48

† Automotive applications.

Interpolation between charts may be necessary for both the I and J factors.

34.6.3 Allowable Stresses

The maximum allowable stresses are based on the properties of the material. They vary with the material, heat treatment, and surface treatment. Table 34.14 gives nominal values for allowable contact stress on gear teeth for commonly used gear materials and heat treatments. Table 34.15 gives nominal values for allowable bending stress in gear teeth for commonly used gear materials and heat treatments.

Carburized case-hardened gears require a core hardness in the range of 260 to 350 H_B (26 to 37 R_C) and a total case depth in the range shown by Fig. 34.49.

The calculated contact stress S_c times a safety factor should be less than the allowable contact stress S_{ac}. The calculated bending stress S_t times a safety factor should be less than the allowable bending stress S_{at}.

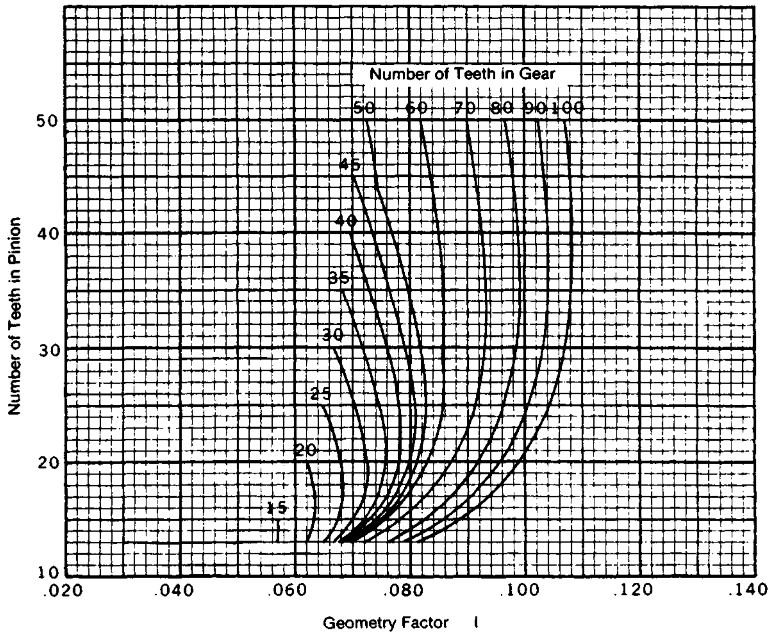

FIGURE 34.19 Geometry factor I for durability of straight-bevel gears with 20° pressure angle and 90° shaft angle.

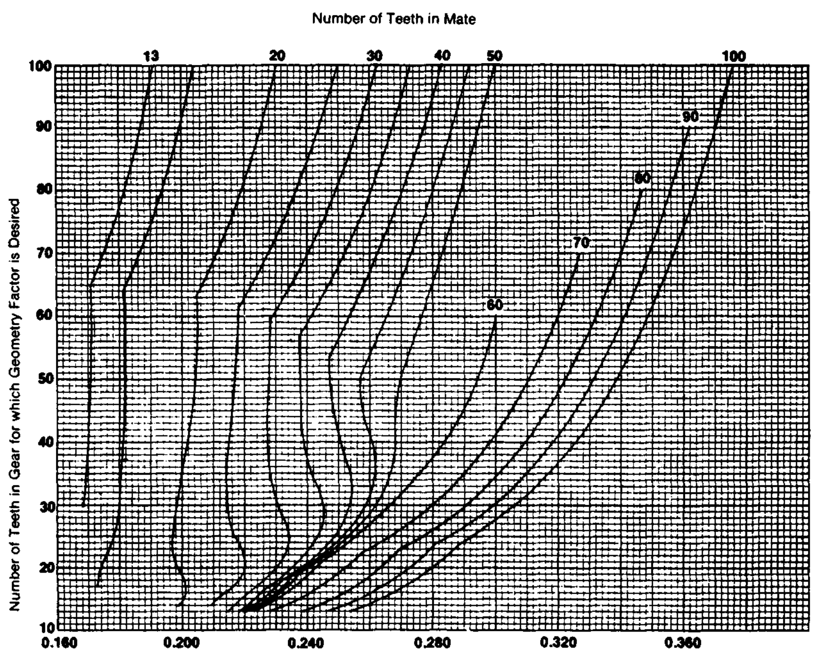

FIGURE 34.20 Geometry factor J for strength of straight-bevel gears with 20° pressure angle and 90° shaft angle.

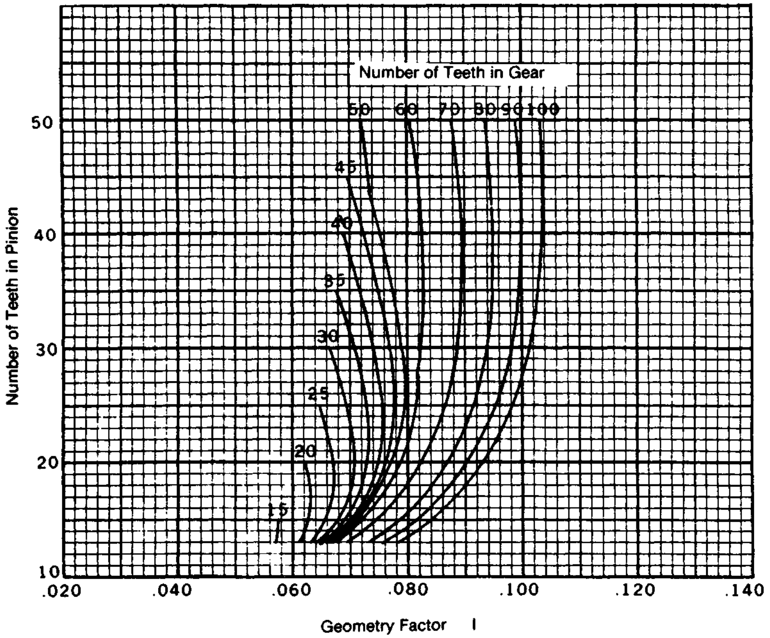

FIGURE 34.21 Geometry factor I for durability of straight-bevel gears with 25° pressure angle and 90° shaft angle.

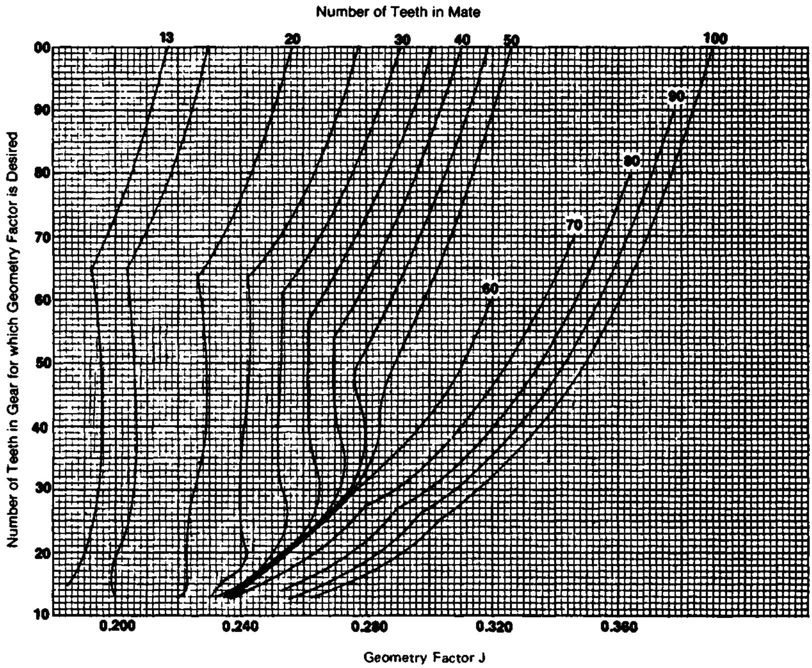

FIGURE 34.22 Geometry factor J for strength of straight-bevel gears with 25° pressure angle and 90° shaft angle.

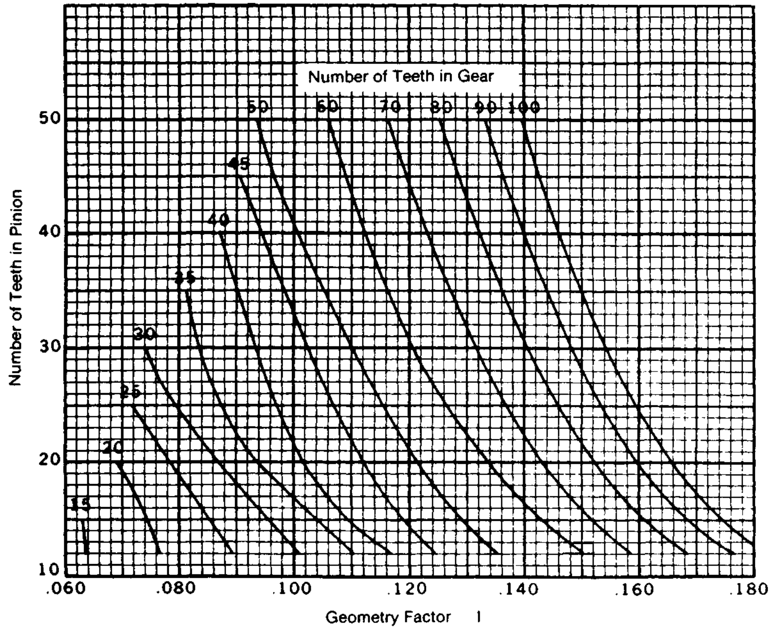

FIGURE 34.23 Geometry factor I for durability of spiral-bevel gears with 20° pressure angle, 35° spiral angle, and 90° shaft angle.

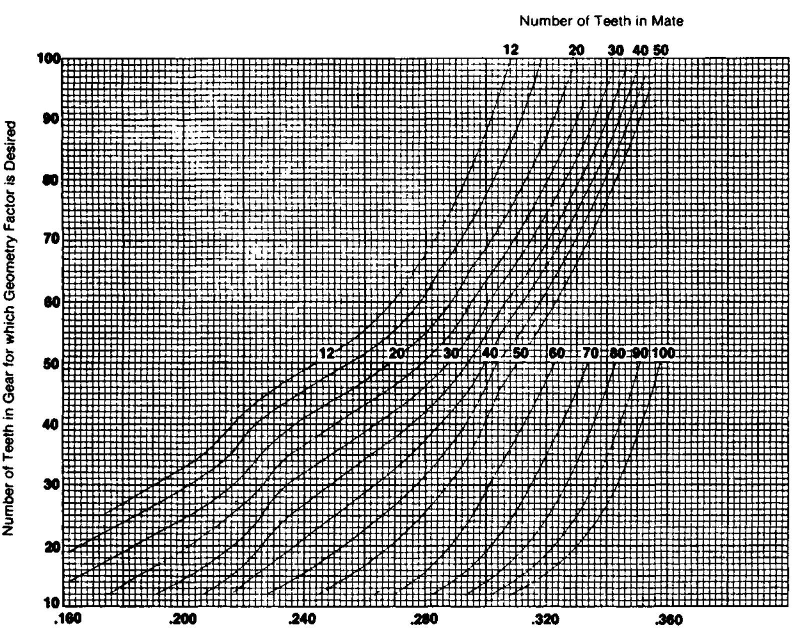

FIGURE 34.24 Geometry factor J for strength of spiral-bevel gears with 20° pressure angle, 35° spiral angle, and 90° shaft angle.

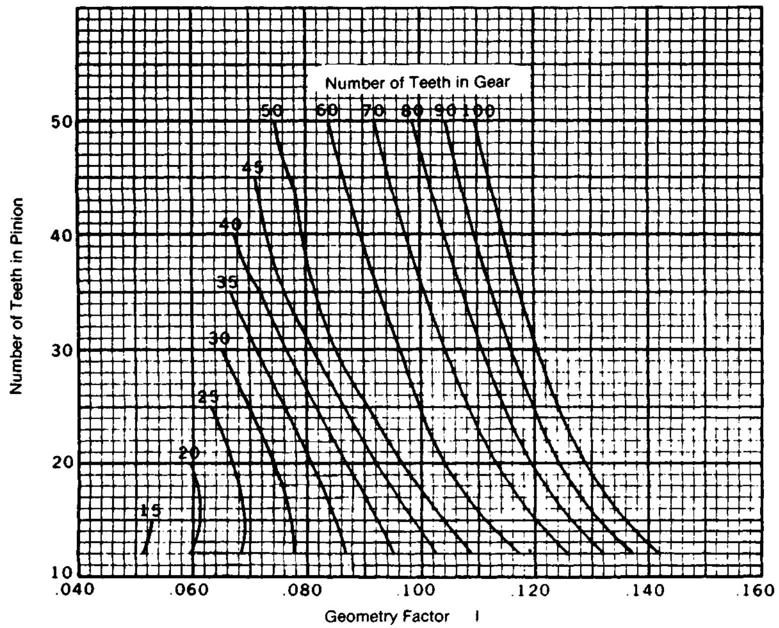

FIGURE 34.25 Geometry factor I for durability of spiral-bevel gears with 20° pressure angle, 25° spiral angle, and 90° shaft angle.

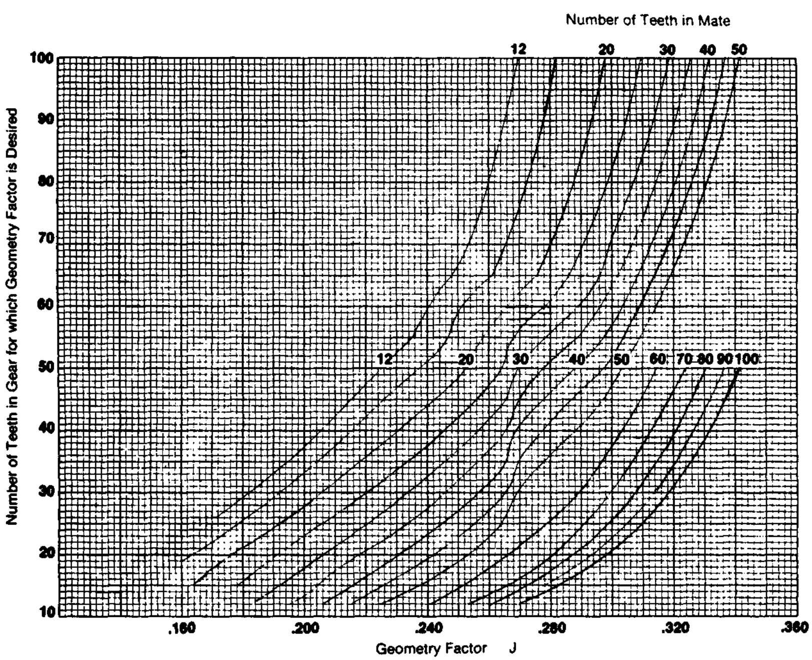

FIGURE 34.26 Geometry factor J for strength of spiral-bevel gears with 20° pressure angle, 25° spiral angle, and 90° shaft angle.

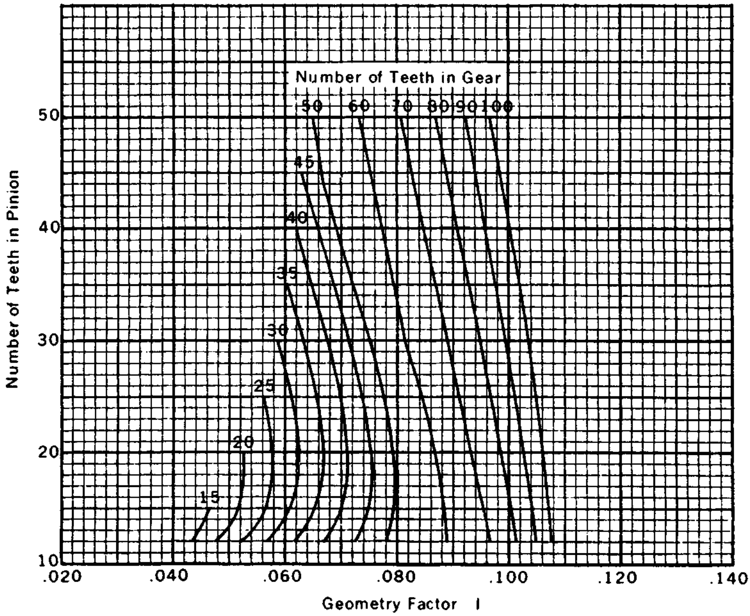

FIGURE 34.27 Geometry factor I for durability of spiral-bevel gears with 20° pressure angle, 15° spiral angle, and 90° shaft angle.

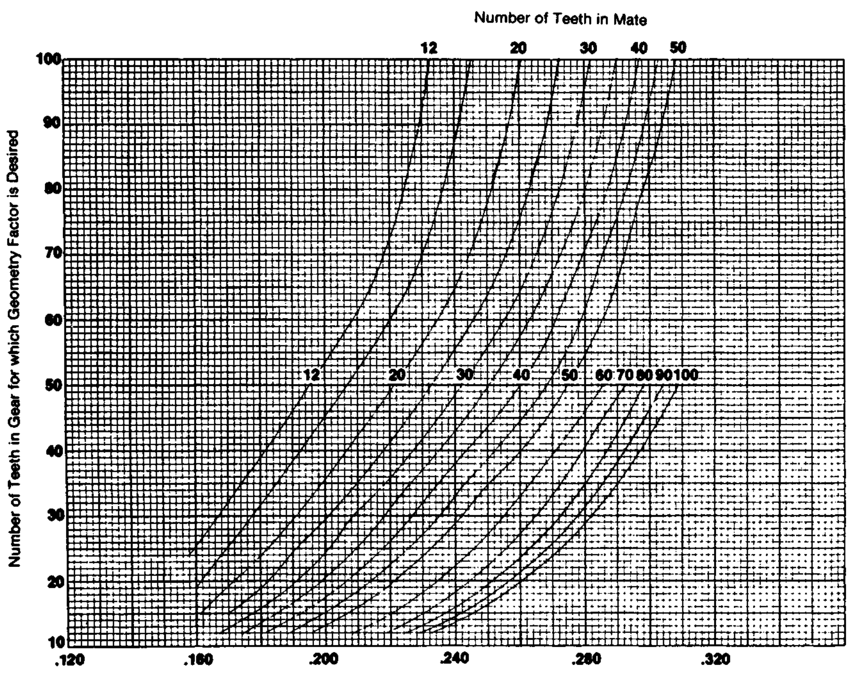

FIGURE 34.28 Geometry factor J for strength of spiral-bevel gears with 20° pressure angle, 15° spiral angle, and 90° shaft angle.

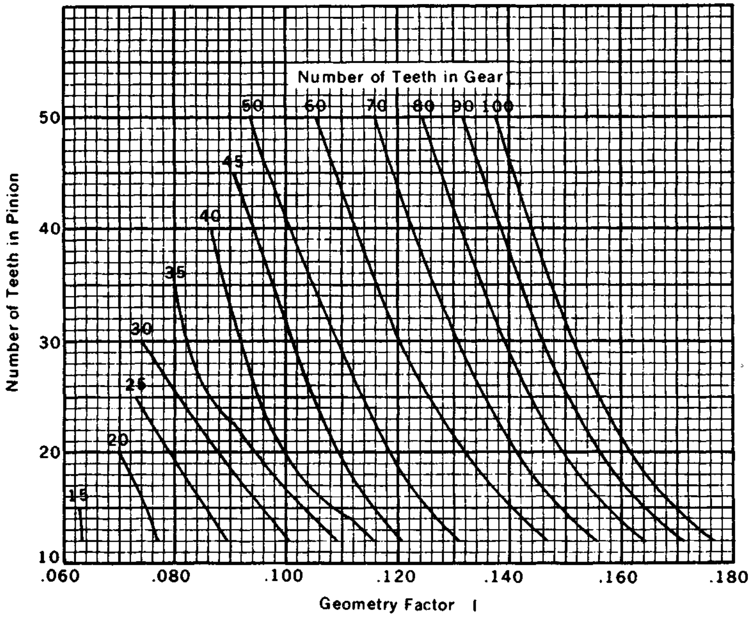

FIGURE 34.29 Geometry factor I for durability of spiral-bevel gears with 25° pressure angle, 35° spiral angle, and 90° shaft angle.

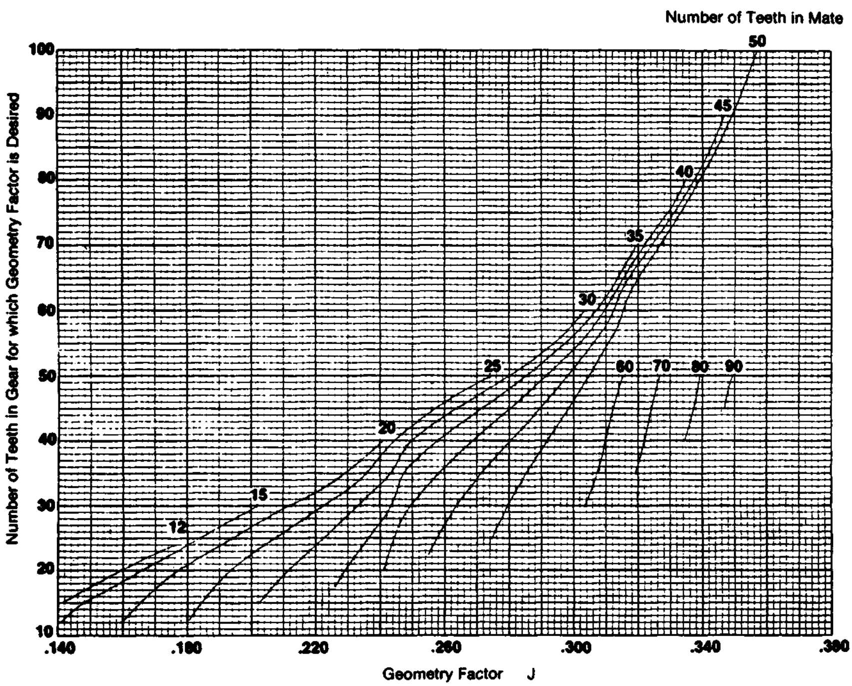

FIGURE 34.30 Geometry factor J for strength of spiral-bevel gears with 25° pressure angle, 35° spiral angle, and 90° shaft angle.

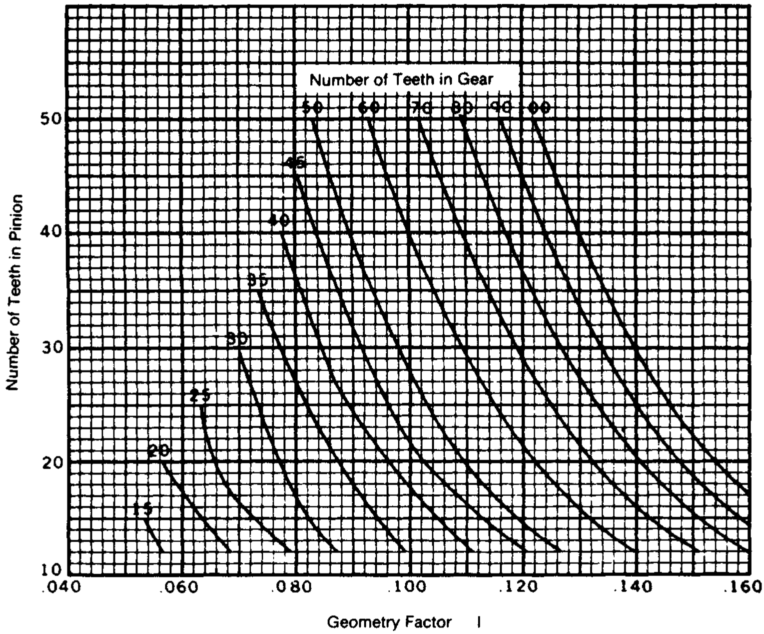

FIGURE 34.31 Geometry factor I for durability of spiral-bevel gears with 20° pressure angle, 35° spiral angle, and 60° shaft angle.

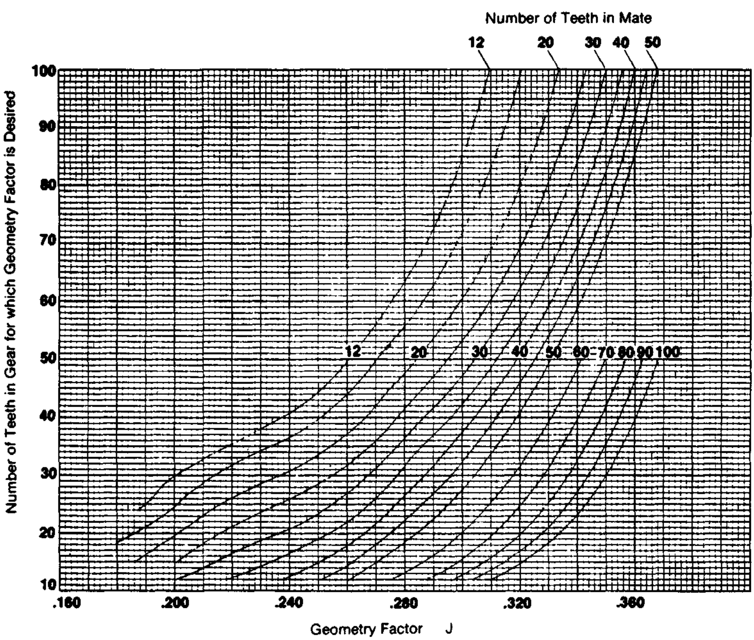

FIGURE 34.32 Geometry factor J for strength of spiral-bevel gears with 20° pressure angle, 35° spiral angle, and 60° shaft angle.

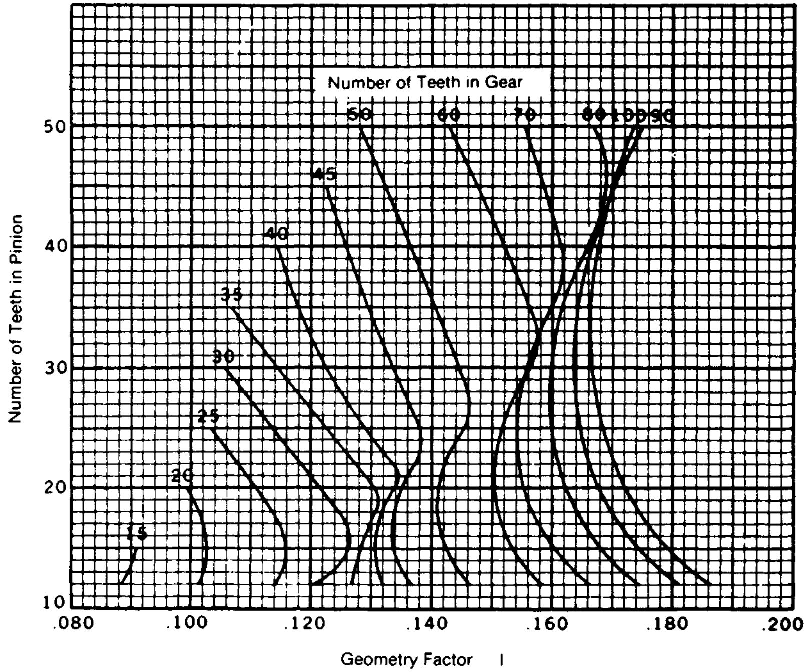

FIGURE 34.33 Geometry factor *I* for durability of spiral-bevel gears with 20° pressure angle, 35° spiral angle, and 120° shaft angle.

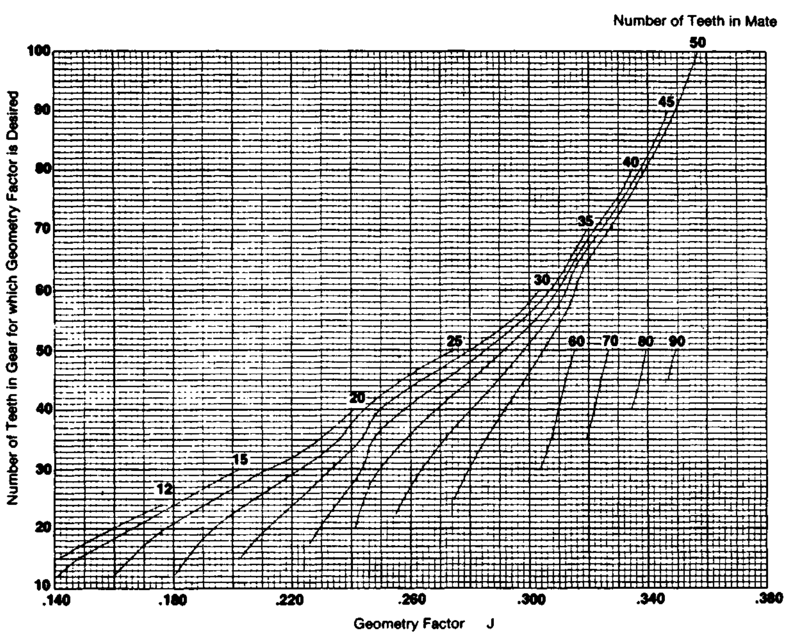

FIGURE 34.34 Geometry factor *J* for strength of spiral-bevel gears with 20° pressure angle, 35° spiral angle, and 120° shaft angle.

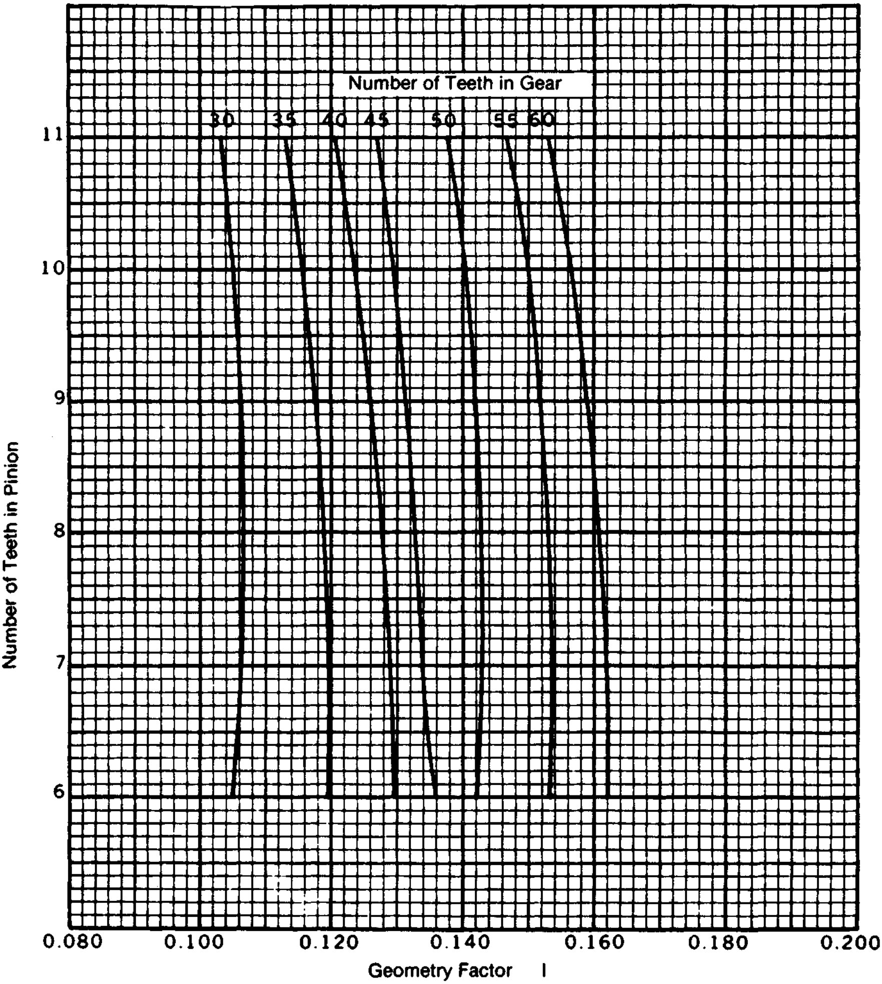

FIGURE 34.35 Geometry factor *I* for durability of automotive spiral-bevel gears with 20° pressure angle, 35° spiral angle, and 90° shaft angle.

34.6.4 Scoring Resistance

Scoring is a temperature-related process in which the surfaces actually tend to weld together. The oil film breaks down, and the tooth surfaces roll and slide on one another, metal against metal. Friction between the surfaces causes heat which reaches the melting point of the tooth material, and scoring results. The factors which could cause scoring are the sliding velocity, surface finish, and load concentrations along with the lubricant temperature, viscosity, and application. But see also Chap. 6. If you follow the recommendations under Sec. 34.7.6 on lubrication and the manufacturer uses acceptable practices in processing the gears, then scoring should not be a problem.

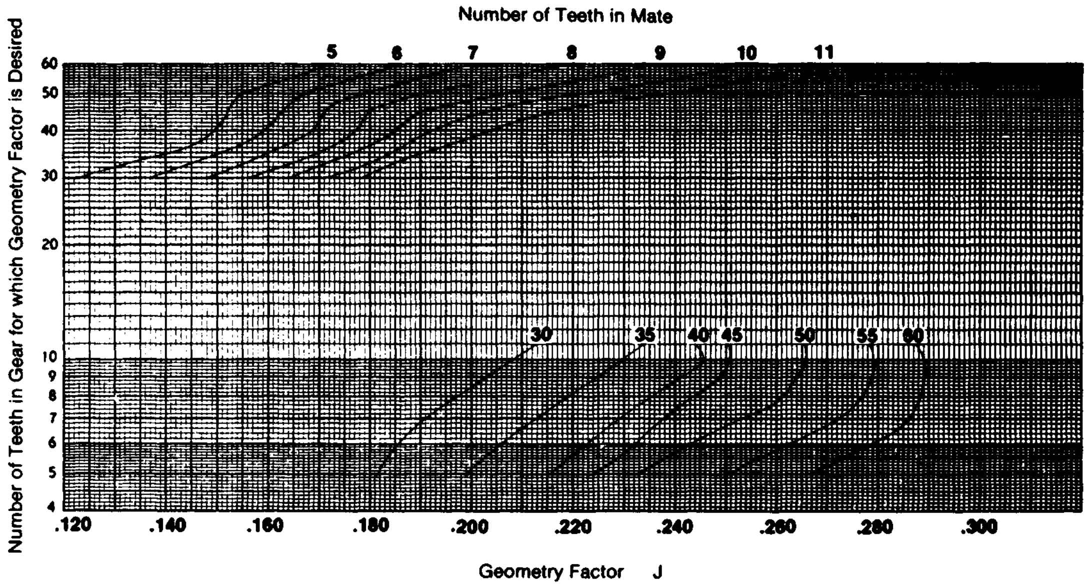

FIGURE 34.36 Geometry factor J for strength of automotive spiral-bevel gears with 20° pressure angle, 35° spiral angle, and 90° shaft angle.

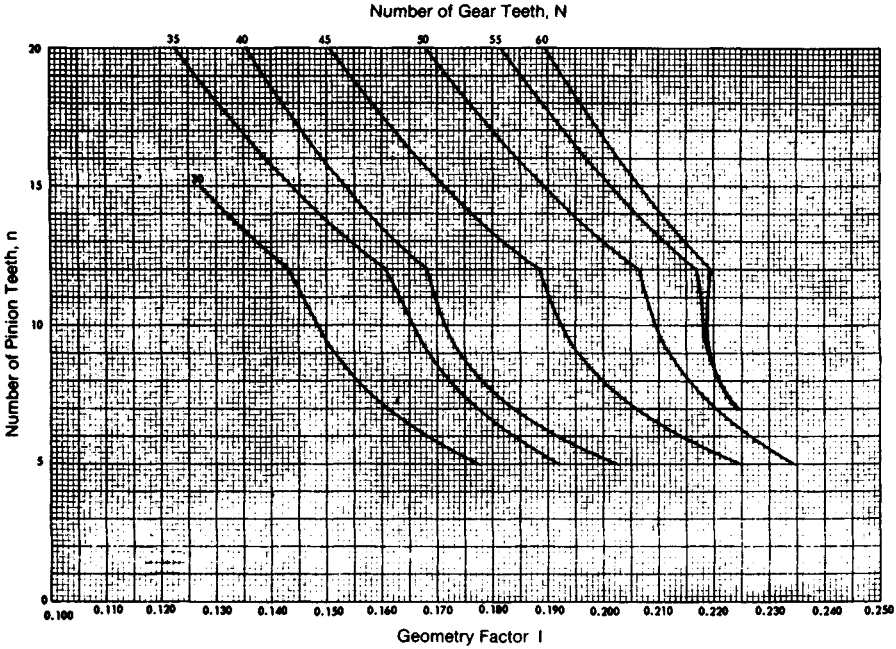

FIGURE 34.37 Geometry factor *I* for hypoid gears with 19° average pressure angle and *E*/*D* ratio of 0.10.

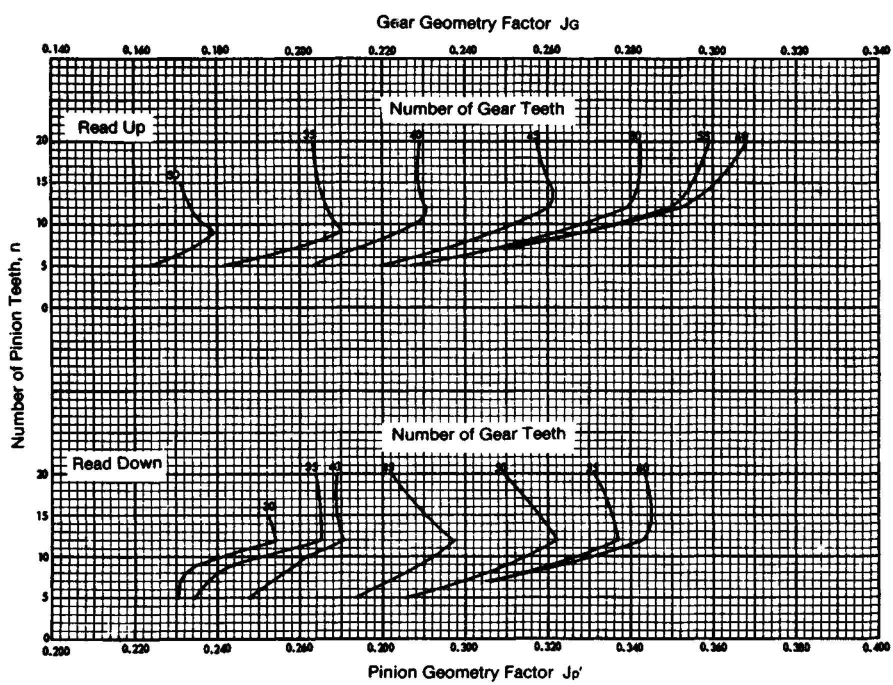

FIGURE 34.38 Geometry factor *J* for strength of hypoid gears with 19° average pressure angle and *E*/*D* ratio of 0.10.

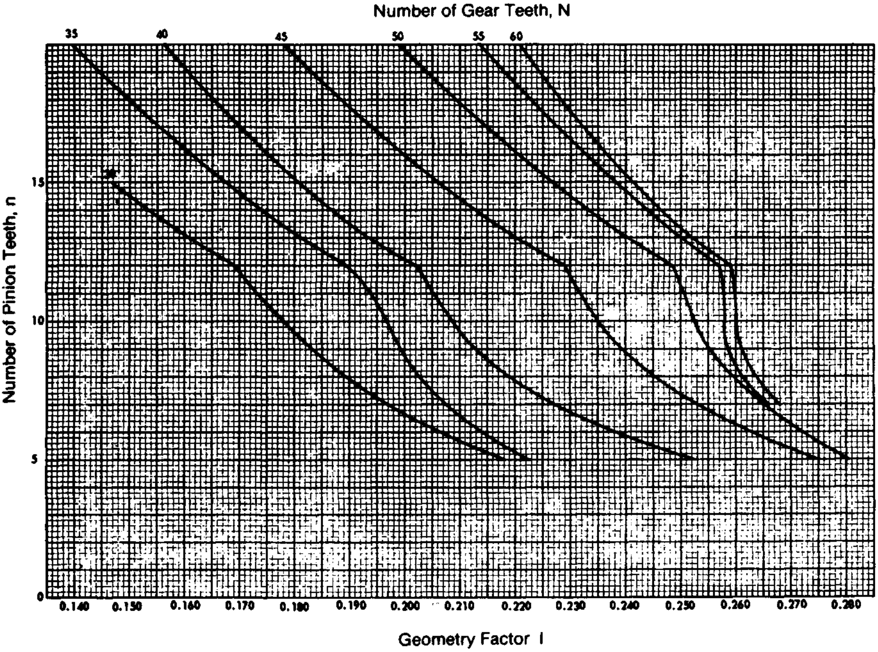

FIGURE 34.39 Geometry factor I for durability of hypoid gears with 19° average pressure angle and E/D ratio of 0.15.

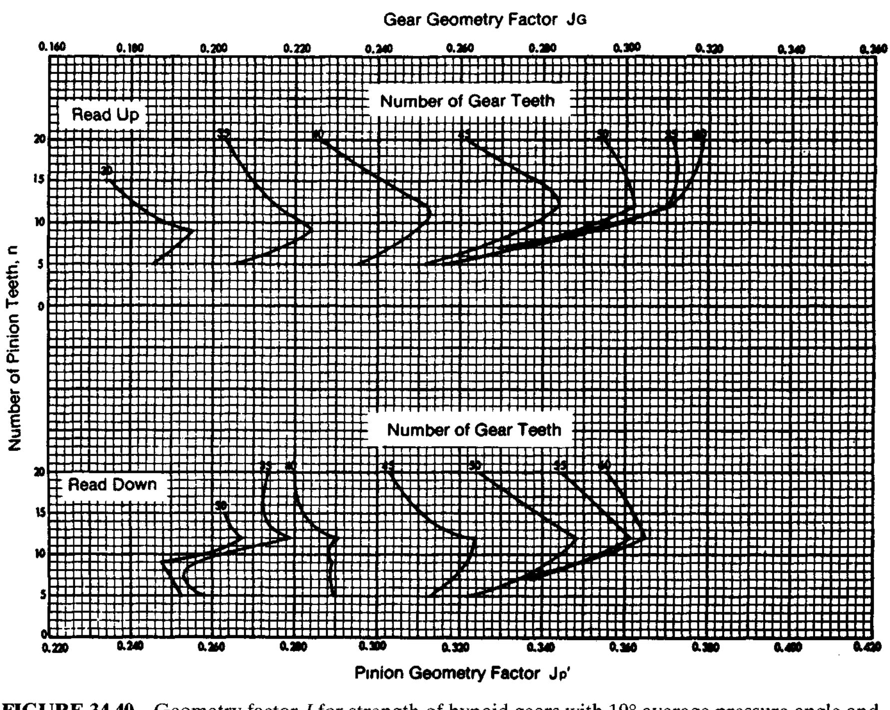

FIGURE 34.40 Geometry factor J for strength of hypoid gears with 19° average pressure angle and E/D ratio of 0.15.

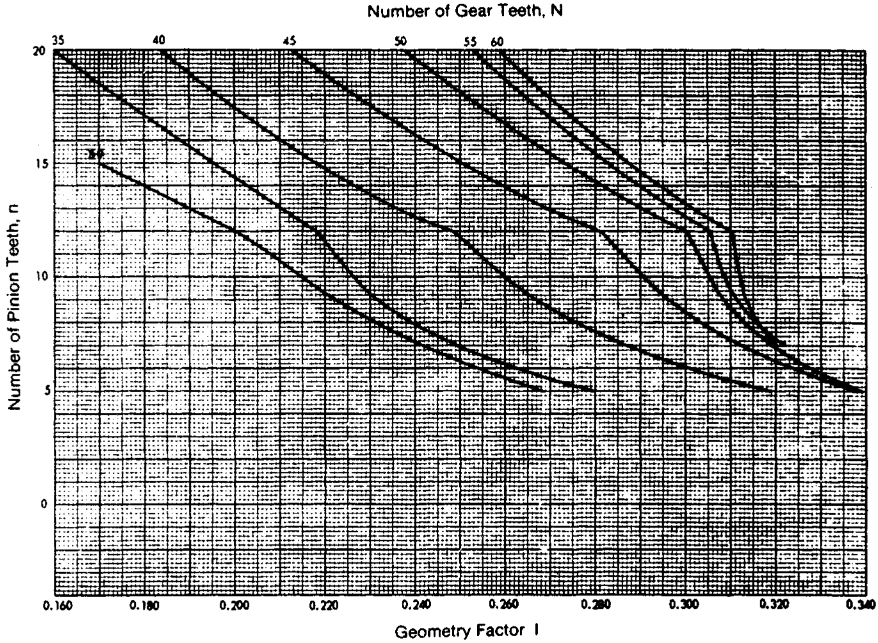

FIGURE 34.41 Geometry factor I for durability of hypoid gears with 19° average pressure angle and E/D ratio of 0.20.

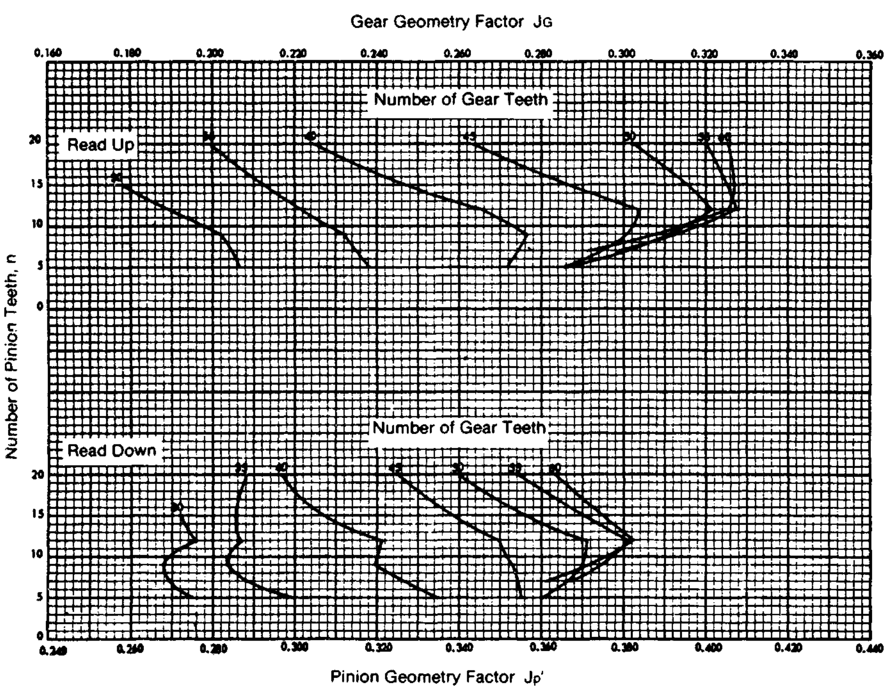

FIGURE 34.42 Geometry factor J for strength of hypoid gears with 19° average pressure angle and E/D ratio of 0.20.

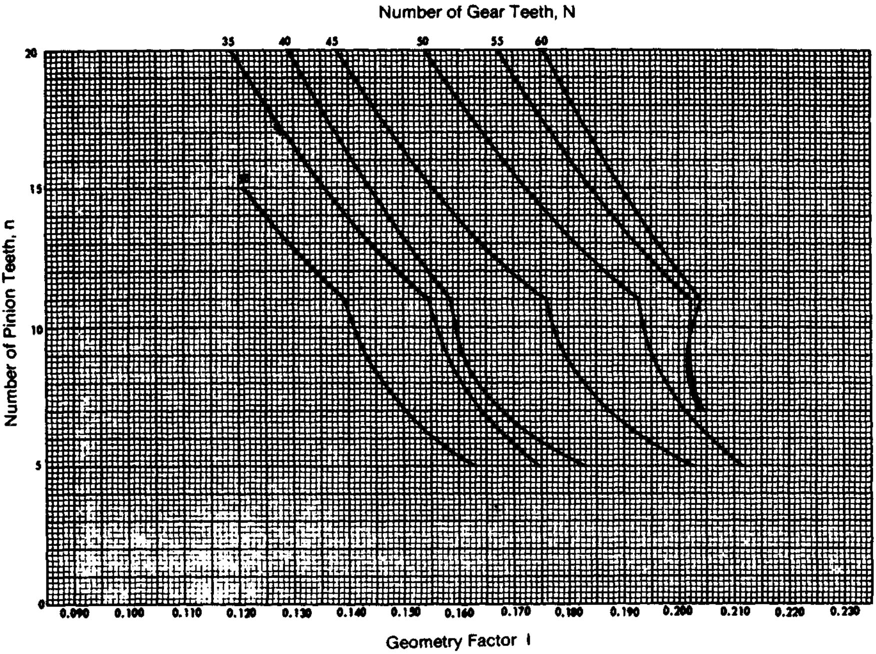

FIGURE 34.43 Geometry factor *I* for durability of hypoid gears with 22½° average pressure angle and *E*/*D* ratio of 0.10.

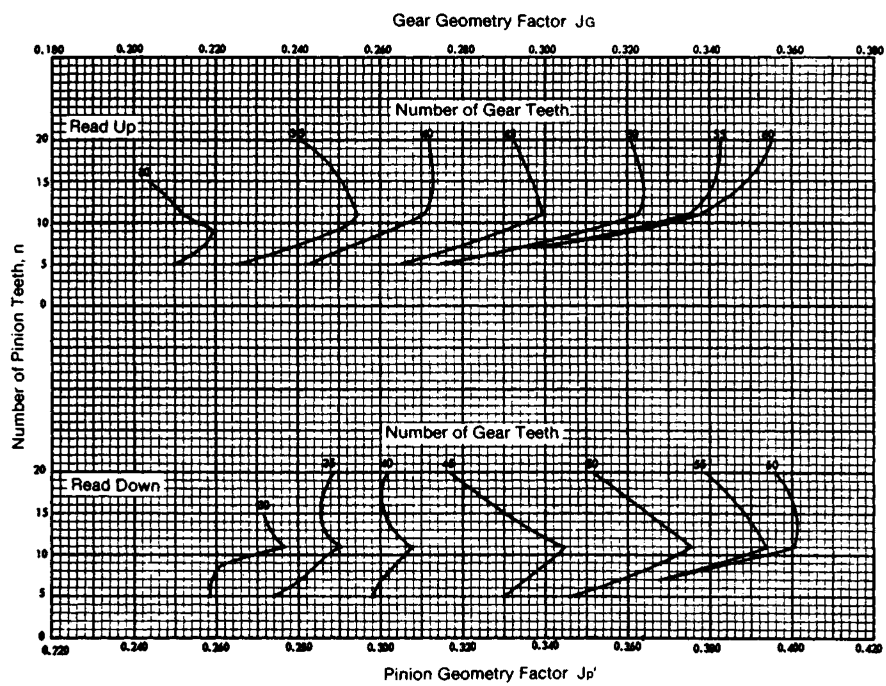

FIGURE 34.44 Geometry factor *J* for strength of hypoid gears with 22½° average pressure angle and *E*/*D* ratio of 0.10.

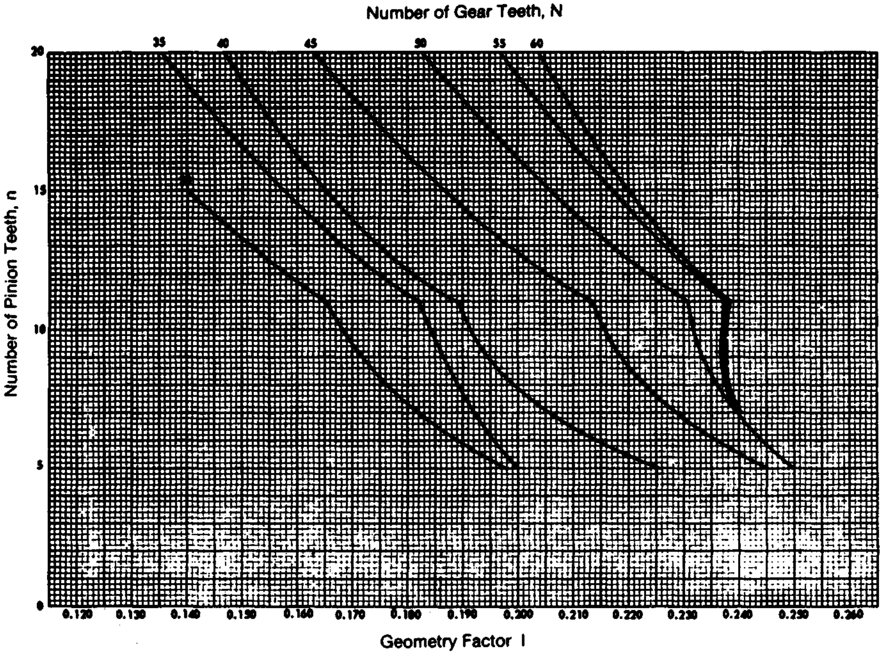

FIGURE 34.45 Geometry factor I for durability of hypoid gears with 22½° average pressure angle and E/D ratio of 0.15.

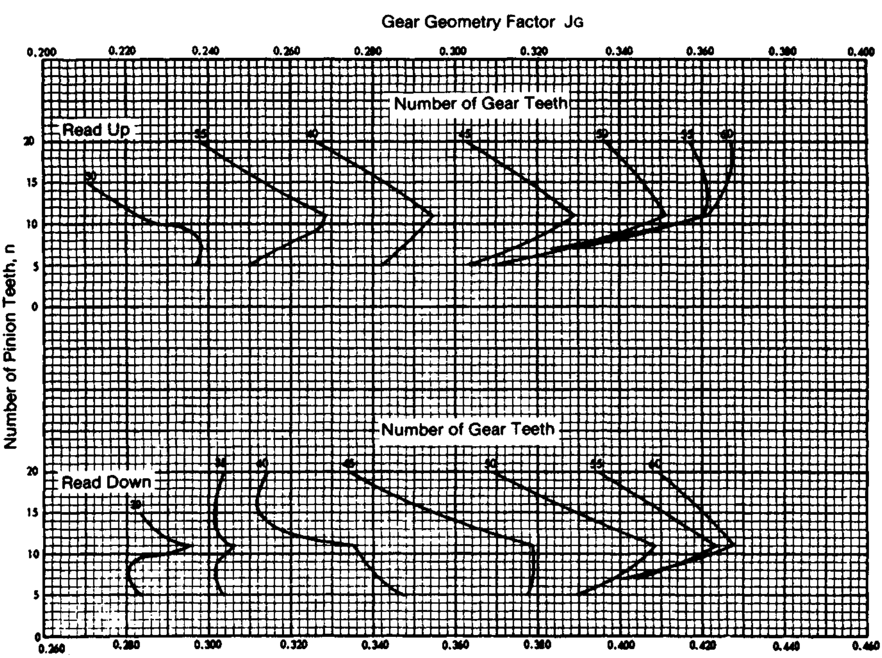

FIGURE 34.46 Geometry factor J for strength of hypoid gears with 22½° average pressure angle and E/D ratio of 0.15.

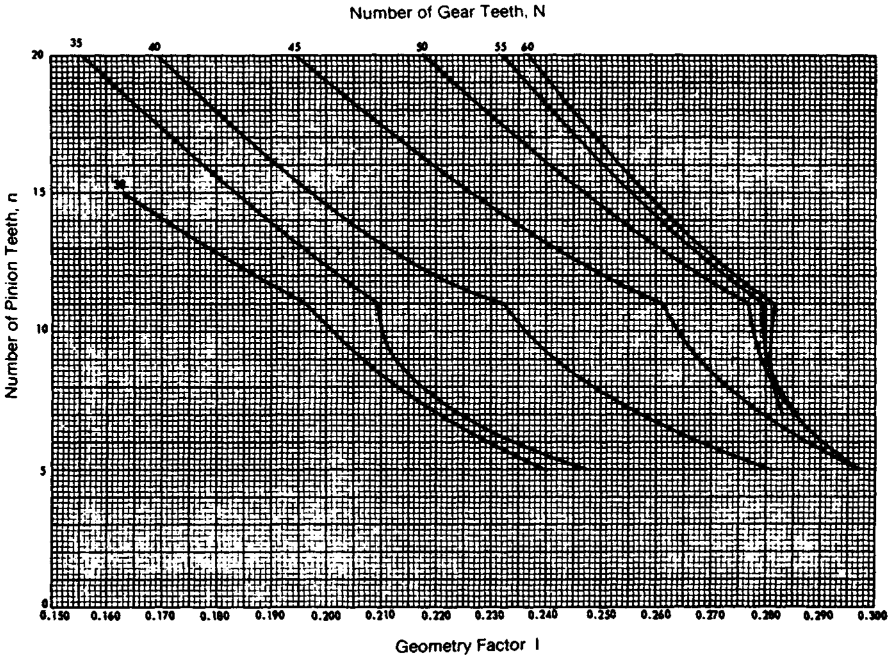

FIGURE 34.47 Geometry factor I for durability of hypoid gears with 22½° average pressure angle and E/D ratio of 0.20.

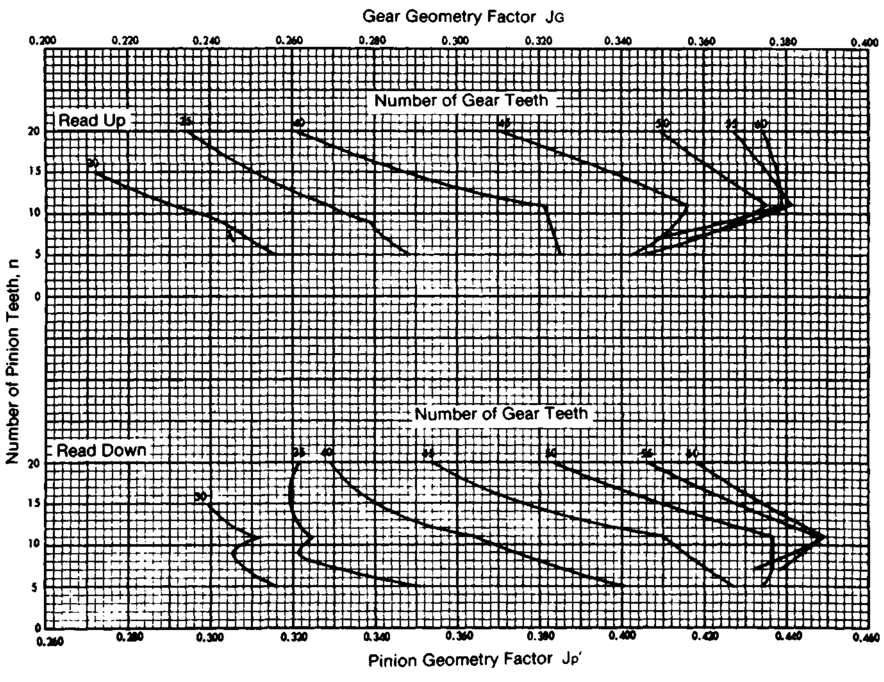

FIGURE 34.48 Geometry factor J for strength of hypoid gears with 22½° average pressure angle and E/D ratio of 0.20.

TABLE 34.14 Allowable Contact Stress S_{ac}

Material	Heat treatment	Minimum hardness		Contact stress S_{ac}, lb/in²
		Brinell	Rockwell C	
Steel	Carburized (case-hardened)		60	250 000
Steel	Carburized (case-hardened)		55	210 000
Steel	Flame- or induction-hardened	500	50	200 000
Steel and nodular iron	Hardened and tempered	400		180 000
Steel	Nitrided		60	180 000
Steel and nodular iron	Hardened and tempered	300		140 000
Steel and nodular iron	Hardened and tempered	180		100 000
Cast iron	As cast	200		80 000
Cast iron	As cast	175		70 000
Cast iron	As cast			60 000

34.7 DESIGN OF MOUNTINGS

The normal load on the tooth surfaces of bevel and hypoid gears may be resolved into two components: one in the direction along the axis of the gear and the other perpendicular to the axis. The direction and magnitude of the normal load depend on the ratio, pressure angle, spiral angle, hand of spiral, and direction of rotation as well as on whether the gear is the driving or driven member.

34.7.1 Hand of Spiral

In general, a left-hand pinion driving clockwise (viewed from the back) tends to move axially away from the cone center; a right-hand pinion tends to move toward the center because of the oblique direction of the curved teeth. If possible, the hand of spiral should be selected so that both the pinion and the gear tend to move out of mesh, which prevents the possibility of tooth wedging because of reduced backlash. Otherwise, the hand of spiral should be selected to give an axial thrust that tends to move the pinion out of mesh. In a reversible drive, there is no choice unless the pair performs a heavier duty in one direction for a greater part of the time.

TABLE 34.15 Allowable Bending Stress S_{at}

Material	Heat treatment	Surface hardness		Bending stress S_{at}, lb/in^2
		Brinell	Rockwell C	
Steel	Carburized (case-hardened)	575–625	55 min.	60 000
Steel	Flame- or induction-hardened (unhardened root fillet)	450–500	50 min.	27 000
Steel	Hardened and tempered	450 min.		50 000
Steel	Hardened and tempered	300 min.		42 000
Steel	Hardened and tempered	180 min.		28 000
Steel	Normalized	140 min.		22 000
Cast iron	As cast	200 min.		13 000
Cast iron	As cast	175 min.		8 500
Cast iron	As cast			5 000

On hypoids when the pinion is below center and to the right (when you are facing the front of the gear), the pinion hand of spiral should always be left-hand. With the pinion above center and to the right, the pinion hand should always be right-hand. See Fig. 34.15.

34.7.2 Tangential Force

The tangential force on a bevel or hypoid gear is given by

$$W_{tG} = \frac{2T_G}{D_m} = \frac{126\,000P}{D_m N} \tag{34.4}$$

where T_G = gear torque, lb · in
P = power, horsepower (hp)
N = speed of gear, r/min

The tangential force on the mating pinion is given by the equation

$$W_{tP} = \frac{W_{tG} \cos \psi_P}{\cos \psi_G} = \frac{2T_P}{d_m} \tag{34.5}$$

where T_P = pinion torque in pound-inches.

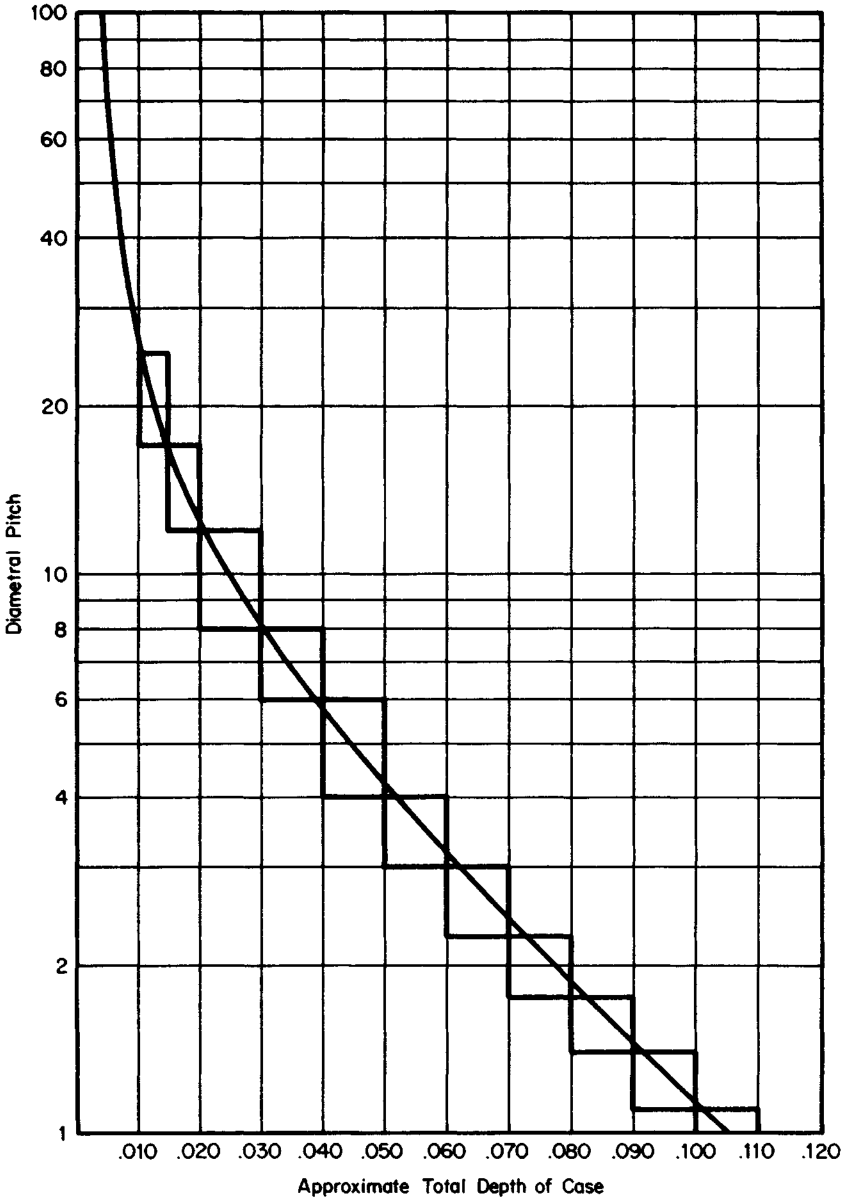

FIGURE 34.49 Diametral pitch versus total case depth. If in doubt, use the greater case depth on ground gears or on short face widths.

34.7.3 Axial Thrust and Radial Separating Forces

The formulas that follow are used to calculate the *axial thrust force* W_x and the *radial separating force* W_R for bevel and hypoid gears. The direction of the pinion (driver) rotation should be viewed from the pinion back.

For a pinion (driver) with a *right-hand* (*RH*) *spiral with clockwise* (*cw*) *rotation* or a *left-hand* (*LH*) *spiral with counterclockwise* (*ccw*) *rotation,* the axial and separating force components *acting on the pinion* are, respectively,

$$W_{xP} = W_{tP} \sec \psi_P \,(\tan \phi \sin \gamma - \sin \psi_P \cos \gamma) \tag{34.6}$$

$$W_{RP} = W_{tP} \sec \psi_P \,(\tan \phi \cos \gamma + \sin \psi_P \sin \gamma) \tag{34.7}$$

For a pinion (driver) with an *LH spiral with cw rotation* or an *RH spiral with ccw rotation,* the force components *acting on the pinion* are, respectively,

$$W_{xP} = W_{tP} \sec \psi_P \,(\tan \phi \sin \gamma + \sin \psi_P \cos \gamma) \tag{34.8}$$

$$W_{RP} = W_{tP} \sec \psi_P \,(\tan \phi \cos \gamma - \sin \psi_P \sin \gamma) \tag{34.9}$$

For a pinion (driver) with an *RH spiral with cw rotation* or an *LH spiral with ccw rotation,* the force components *acting on the gear* (driven) are, respectively,

$$W_{xG} = W_{tG} \sec \psi_G \,(\tan \phi \sin \Gamma + \sin \psi_G \cos \Gamma) \tag{34.10}$$

$$W_{RG} = W_{tG} \sec \psi_G \,(\tan \phi \cos \Gamma - \sin \psi_G \sin \Gamma) \tag{34.11}$$

For a pinion (driver) with an *LH spiral and cw rotation* or an *RH spiral with ccw rotation,* the force components *acting on the gear* are, respectively,

$$W_{xG} = W_{tG} \sec \psi_G \,(\tan \phi \sin \Gamma - \sin \psi_G \cos \Gamma) \tag{34.12}$$

$$W_{RG} = W_{tG} \sec \psi_G \,(\tan \phi \cos \Gamma + \sin \psi_G \sin \Gamma) \tag{34.13}$$

These equations apply to straight-bevel, Zerol bevel, spiral-bevel, and hypoid gears. When you use them for hypoid gears, be sure that the pressure angle corresponds to the driving face of the pinion tooth.

A plus sign for Eqs. (34.6), (34.8), (34.10), and (34.12) indicates that the direction of the axial thrust is *outward,* or away from the cone center. Thus a minus sign indicates that the direction of the axial thrust is *inward,* or toward the cone center.

A plus sign for Eqs. (34.7), (34.9), (34.11), and (34.13) indicates that the direction of the *separating* force is *away* from the mating gear. So a minus sign indicates an *attracting* force *toward* the mating member.

Example. A hypoid-gear set consists of an 11-tooth pinion with LH spiral and ccw rotation driving a 45-tooth gear. Data for the gear are as follows: 4.286 diametral pitch, 8.965-inch (in) mean diameter, 70.03° pitch angle, 31.48° spiral angle, and 30×10^3 lb · in torque. Pinion data are these: 1.500-in offset, 2.905-in mean diameter, concave pressure angle 18.13°, convex pressure angle 21.87°, pitch angle 19.02°, and spiral angle 50°. Determine the force components and their directions for each member of the set.

Solution. From Eq. (34.4) we find the tangential load on the gear to be

$$W_{tG} = \frac{2T_G}{D_m} = \frac{2(30 \times 10^3)}{8.965} = 6693 \text{ lb}$$

Since the pinion has LH spiral angle and rotates ccw, Eqs. (34.10) and (34.11) apply for the gear. Thus

$$\begin{aligned} W_{xG} &= W_{tG} \sec \psi_G (\tan \phi \sin \Gamma + \sin \psi_G \cos \Gamma) \\ &= 6693 \sec 31.48 (\tan 18.13° \sin 70.03° + \sin 31.48° \cos 70.03°) \\ &= 3814 \text{ lb} \end{aligned}$$

Substituting the same values and angles into Eq. (34.11) gives $W_{RG} = -2974$ lb. Thus the thrust is outward, and the separating force is toward the mating member.

Next we find the tangential load on the pinion from Eq. (34.5):

$$W_{tP} = \frac{W_{tG} \cos \psi_P}{\cos \psi_G} = \frac{6693 \cos 50°}{\cos 31.48°} = 5045 \text{ lb}$$

Equations (34.6) and (34.7) apply to the pinion:

$$\begin{aligned} W_{xP} &= W_{tP} \sec \psi_P (\tan \phi \sin \gamma - \sin \psi_P \cos \gamma) \\ &= 5045 \sec 50° (\tan 18.13° \sin 19.02° - \sin 50° \cos 19.02°) \\ &= -4846 \text{ lb} \end{aligned}$$

In a similar manner, Eq. (34.7) gives $W_{RP} = 4389$ lb. Thus the axial thrust is inward, and the separating force is away from the gear.

34.7.4 Bearing Loads

The bearings selected must be adequate to support the axial forces W_x for both directions of rotation and for the load conditions on both sides of the teeth.

Radial forces are transmitted indirectly through moment arms to the bearings. The radial bearing loads are derived from the gear separating force, the gear tangential force, and the gear thrust couple, along with the type of mounting and the bearing position.

34.7.5 Types of Mountings

Two types of mountings are generally used: *overhung,* where both bearings are located on the shaft behind the gear, and *straddle,* where one bearing is on either side of the gear. Because of the stiffer configuration, straddle mountings are generally used for highly loaded gears.

34.7.6 Lubrication

The lubrication system for a bevel- or hypoid-gear drive should sufficiently lubricate and adequately cool the gears and bearings. Splash lubrication is generally satisfactory for applications up to peripheral speeds of 2000 ft/min. The oil level should cover the full face of the lowest gear, and the quantity of oil should be sufficient to maintain the oil temperature within recommended limits.

Pressure lubrication is recommended for velocities above 2000 ft/min. The jets should be located to direct the stream to cover the full length of the teeth of both members, preferably close to the mesh point on the leaning side.

Experience has shown that an oil flow of 0.07 to 1.0 gallons per minute (gal/min) per 100 hp will result in an oil temperature rise of approximately 10°F.

Extreme-pressure (EP) lubricants are recommended for hypoid gears and for spiral-bevel gears which are subject to extreme conditions of shock, severe starting conditions, or heavy loads. The lubrication system should be fully protected against contamination by moisture or dirt. For continuous operation at temperatures above 160°F, the lubricants should be approved by the lubricant manufacturer.

In general, for a splash lubrication, an SAE 80 or 90 gear oil should be satisfactory. For a circulating system with an oil spray lubrication, SAE 20 or 30 should be satisfactory. AGMA "Specifications on Lubrication of Enclosed and Open Gearing" is a recommended guide to the type and grade of oil for various operating conditions.

34.7.7 Loaded Contact Check

With highly stressed bevel- and hypoid-gear applications such as aircraft and automotive, it is normal practice to perform a loaded contact check with the gear set assembled in its mountings. A brake load is applied to the output shafts, and the pinion member is rotated slowly at approximately 15 r/min. A marking compound is applied to the pinion and gear teeth to permit observation of the tooth contact pattern at the desired load conditions. The purpose of this test is to evaluate the rigidity of the mountings and ensure that the contact pattern remains within the tooth boundaries under all load conditions. Indicators can be mounted at various positions under load. An analysis of these data can result in modifications of the mounting design or contact pattern to ensure that the contact pattern does not reach the tooth boundaries at operating loads. This will eliminate an edge contact condition which can cause noise or premature failure of the gear teeth.

34.8 COMPUTER-AIDED DESIGN

34.8.1 Computer Timesharing

A computer timesharing service is available to assist you with gear-tooth design, strength calculations, gear-tooth geometry analysis, gear manufacturing, and inspection data for bevel and hypoid gears. Contact

Application Engineering Department
Gleason Machine Division
1000 University Avenue
Rochester, New York 14692

34.8.2 Design Calculating Services

The Gleason Machine Division offers a calculating service which may be used as an alternative to the computer timesharing service mentioned earlier, when you require a computer analysis of the gear-tooth design.

34.8.3 Available Computer Programs

The following computer programs are available from the Gleason Machine Division to assist you with a gear-tooth design analysis:

1. *Dimension Sheet* Calculation of the basic tooth geometry, contact ratios, stress data, bearing thrust loads, and profile sliding velocities.
2. *Summary* Calculation of cutting and grinding machine setup data to produce the desired tooth geometry.
3. *Tooth Contact Analysis* A special analysis program that determines the tooth contact pattern and transmission motion errors based on specified cutting tools and gear-tooth geometry. Figure 34.50 illustrates a typical *tooth contact analysis.*
4. *Undercut Check* Calculation of the location of undercut lengthwise along the tooth, along with the depth and angle of undercut relative to the tooth profile.
5. *Loaded Tooth Contact Analysis* An analysis and plot of tooth contact pattern and transmission errors as a function of gear torque. Deflections of the gear mountings may also be considered with this analysis.
6. *Finite-Element Analysis* Detailed stress data calculated based on a three-dimensional finite-element stress model which considers exact gear-tooth geometry based on cutting tool specifications, machine setup, and generating motions and mounting deflections.

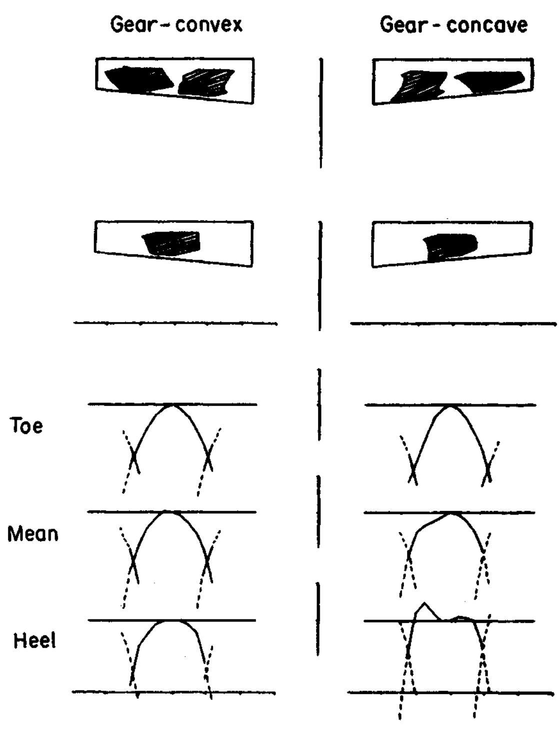

FIGURE 34.50 Typical tooth analysis contact graph.